

Calculus Warm Up #5-1

Test for convergence using a test from the side board.
State the test(s) used and give a complete conclusion.

$$\sum_{n=1}^{\infty} \left| \frac{\cos n}{2^n} \right|$$

HW Questions: p. 604

$$\begin{aligned} 9) \sum_{n=0}^{\infty} \frac{(-1)^n 2^n}{n!} & \quad \lim_{n \rightarrow \infty} \frac{2^n}{n!} \\ & = \lim_{n \rightarrow \infty} \left[\frac{2^{n+1}}{(n+1)!} \cdot \frac{n!}{2^n} \right] \end{aligned}$$

$$\begin{aligned} \frac{n!}{(n+1)!} & = \frac{n(n-1)(n-2)(n-3)(n-4) \dots}{(n+1)(n+1-1)(n+1-2)(n+1-3)(n+1-4) \dots} \\ & = \frac{\cancel{n}(\cancel{n-1})(\cancel{n-2})(\cancel{n-3})(\cancel{n-4}) \dots}{(n+1)\cancel{n}(\cancel{n-1})(\cancel{n-2})(\cancel{n-3}) \dots} \\ & = \frac{1}{n+1} \end{aligned}$$

$$\begin{aligned}
 15) \sum_{n=0}^{\infty} \frac{3^n}{(n+1)^n} & \lim_{n \rightarrow \infty} \left[\frac{3^{n+1}}{(n+2)^{n+1}} \cdot \frac{(n+1)^n}{3^n} \right] \\
 & = \lim_{n \rightarrow \infty} \left(\frac{3}{n+2} \right) \cdot \lim_{n \rightarrow \infty} \left(\frac{n+1}{n+2} \right)^n \\
 & = 0 \cdot \frac{1}{e} \\
 & = 0 < 1 \quad \text{converges}
 \end{aligned}$$

★ See next slide for how to evaluate $\lim_{n \rightarrow \infty} \left(\frac{n+1}{n+2} \right)^n$

$$\begin{aligned}
 \lim_{n \rightarrow \infty} \left(\frac{n+1}{n+2} \right)^n & \quad \text{let } y = \left(\frac{n+1}{n+2} \right)^n \\
 \ln y & = n \ln \left(\frac{n+1}{n+2} \right) \\
 \ln y & = \lim_{n \rightarrow \infty} \left[\frac{\ln(n+1) - \ln(n+2)}{\frac{1}{n}} \right] = \frac{0}{0} \\
 \ln y & = \lim_{n \rightarrow \infty} \frac{\left(\frac{1}{n+1} - \frac{1}{n+2} \right)}{-\frac{1}{n^2}} \\
 \ln y & = \lim_{n \rightarrow \infty} \left(\frac{n+2 - (n+1)}{n^2 + 3n + 2} \cdot \frac{-n^2}{1} \right) \\
 \ln y & = \lim_{n \rightarrow \infty} \frac{-n^2}{n^2 + 3n + 2} \\
 \ln y & = -1 \\
 y & = \frac{1}{e} \quad \rightarrow \quad \therefore \lim_{n \rightarrow \infty} \left(\frac{n+1}{n+2} \right)^n = \frac{1}{e}
 \end{aligned}$$

$$\begin{aligned}
 17) \quad \sum_{n=0}^{\infty} \frac{4^n}{3^{n+1}} & \quad \lim_{n \rightarrow \infty} \left[\frac{4^{n+1}}{3^{n+1} + 1} \cdot \frac{3^n + 1}{4^n} \right] \\
 &= \lim_{n \rightarrow \infty} \frac{4(3^n + 1)}{3^{n+1} + 1} \cdot \frac{\frac{1}{3^n}}{\frac{1}{3^n}} \\
 &= \lim_{n \rightarrow \infty} \left(\frac{4(1 + \frac{1}{3^n})}{3 + \frac{1}{3^n}} \right) \\
 &= \frac{4}{3} > 1 \quad \text{diverges.}
 \end{aligned}$$

$\frac{1}{3^n} \cdot \frac{3^{n+1}}{1}$

$$\begin{aligned}
 19) \quad \sum_{n=0}^{\infty} \frac{(-1)^{n+1} n!}{1 \cdot 3 \cdot 5 \cdots (2n+1)} \\
 \lim_{n \rightarrow \infty} \left(\frac{(n+1)!}{\cancel{1 \cdot 3 \cdot 5 \cdots (2n+1)} (2(n+1)+1)} \cdot \frac{\cancel{1 \cdot 3 \cdot 5 \cdots (2n+1)}}{n!} \right) \\
 \lim_{n \rightarrow \infty} \frac{n+1}{2n+3} = \frac{1}{2} < 1 \\
 \text{converges.}
 \end{aligned}$$

Classwork: BC MC - A Practice test

*Things we still need to learn:

#7, 10, 17, 26 (#13: derivatives of parametric equations. Try it!)

check answers:

1. D	6. B	11. B	16. E	21. E	26. C
2. C	7. B	12. D	17. D	22. A	27. E
3. C	8. D	13. B	18. B	23. B	28. A
4. E	9. A	14. C	19. D	24. D	
5. D	10. D	15. E	20. E	25. E	

MC - A

$$2) p(0) = 4 - \int_0^{\pi/2} \sin(2t) dt$$

$$= 4 - \int_0^{\pi/2} 2 \sin t \cos t dt$$

$$= 4 - 2 \left[\frac{(\sin t)^2}{2} \right]_0^{\pi/2}$$

$$= 4 - \left[\left(\sin \frac{\pi}{2} \right)^2 - \left(\sin 0 \right)^2 \right]$$

$$= 4 - 1$$

$$= 3$$

C

$$u = \sin t$$

$$du = \cos t dt$$

$$4) \lim_{h \rightarrow 0} \frac{\tan(0+h) - \tan(0)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{\tan h - 0}{h} = \frac{0}{0} \quad \text{so L'Hôpital}$$

$$= \lim_{h \rightarrow 0} \sec^2 h$$

$$\textcircled{E} \sec^2 x$$

$$14) \int \frac{8}{(x+2)(x-2)} \quad \frac{8}{(x+2)(x-2)} = \frac{A}{x+2} + \frac{B}{x-2}$$

$$8 = A(x-2) + B(x+2)$$

$$\text{for } x=2 \quad 8 = B(4) \rightarrow \textcircled{B=2}$$

$$\text{for } x=-2 \quad 8 = A(-4) \rightarrow \textcircled{A=-2}$$

$$= \int \frac{-2}{x+2} dx + \int \frac{2}{x-2} dx$$

$$= -2 \ln|x+2| + 2 \ln|x-2| + C$$

$$= 2 \left[\ln|x-2| - \ln|x+2| \right] + C$$

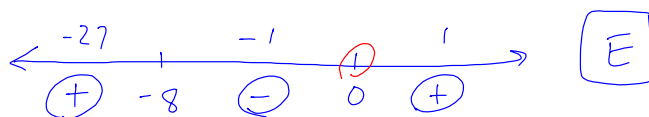
$$= 2 \ln \left| \frac{x-2}{x+2} \right| + C$$

$$\boxed{C}$$

21) where $f' = 0$ or undef. and goes from - to +

$$f'(x) = \frac{6}{\sqrt[3]{x}} + 3 \quad f' = 0 @ x = -8$$

$$\text{undef @ } x = 0$$



22) $\int x f'(x) dx$ let $u = x$ $dv = f'(x) dx$
 $du = 1 dx$ $v = f(x)$

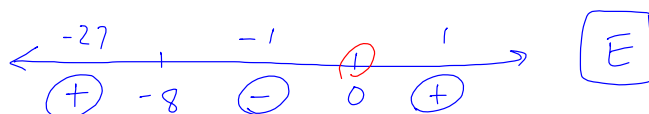
$$\int x f'(x) dx = x \cdot f(x) - \int f(x) dx$$

@ $x = 4 \rightarrow \int_0^4 x \cdot f'(x) dx = 4 \cdot f(4) - \int_0^4 f(x) dx$
 $= 4(-3) - 8$
 $= -20$ A

21) where $f' = 0$ or undef. and goes from - to +

$$f'(x) = \frac{6}{\sqrt[3]{x}} + 3 \quad f' = 0 @ x = -8$$

$$\text{undef @ } x = 0$$



22) $\int x f'(x) dx$ let $u = x$ $dv = f'(x) dx$
 $du = 1 dx$ $v = f(x)$

$$\int x f'(x) dx = x \cdot f(x) - \int f(x) dx$$

@ $x = 4 \rightarrow \int_0^4 x \cdot f'(x) dx = 4 \cdot f(4) - \int_0^4 f(x) dx$
 $= 4(-3) - 8$
 $= -20$ A

$$27) \quad f'(x) = \frac{d}{dx} \int_4^{2x} \sqrt{t^2 - t} \, dt$$

$$= \sqrt{(2x)^2 - (2x)} \cdot \frac{d}{dx}[2x]$$

$$f'(x) = 2\sqrt{4x^2 - 2x}$$

$$f'(2) = 2\sqrt{12}$$

E

Classwork & Homework: BC Practice MC part B

Still need to learn:

78, 86, 87, 90

MC part B answers:

		86. E
76. B	81. A	87. B
77. B	82. C	88. C
78. D	83. D	89. B
79. E	84. C	90. D
80. B	85. B	91. B
		92. C